EXAMINATIONS COUNCIL OF ZAMBIA

Joint Examination for the School Certificate and General Certificate of Education Ordinary Level

ADDITIONAL MATHEMATICS 4030/2
PAPER 2

Thursday 15 NOVEMBER 2012

Additional materials:
Answer Booklet
Mathematical tables
Electronic calculators
Graph paper (1 sheet)

TIME: 2 hours 30 minutes

INSTRUCTIONS TO CANDIDATES

Write your name, centre number and candidate number in the spaces on the separate answer booklet provided.

There are twelve (12) questions in this paper. Answer all questions.

Write your answers on the Answer Booklet provided.

If you use more than one Answer Booklet, fasten the Answer Booklets together.

Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION FOR CANDIDATES

The number of marks is shown in brackets [ ] at the end of each question or part question. The total number of marks for this paper is 100.

The use of a non programmable electronic calculator is expected, where appropriate.

Cell phones should not be brought in the examination room.

Check the formulae overleaf

This question paper consists of 5 printed pages.
Mathematical Formulae

1 ALGEBRA

Quadratic Equation
For the equation \( ax^2 + bx + c = 0 \),
\[
x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}
\]

2 SERIES

Arithmetic \( S_n = \frac{1}{2} n \{2a + (n - 1) d\} \)

Geometric \( S_n = \frac{a(1 - r^n)}{1 - r} \) \( (r \neq 1) \)
\[
S_\infty = \frac{a}{1 - r} \text{ for } |r| < 1
\]

3 TRIGONOMETRY

Identities
\( \sin^2 A + \cos^2 A = 1. \)
\( \sec^2 A = 1 + \tan^2 A. \)
\( \cosec^2 A = 1 + \cot^2 A. \)

Formula for \( \Delta \) ABC
\[
\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}
\]
\( a^2 = b^2 + c^2 - 2bc \cos A. \)
\( \Delta = \frac{1}{2} bc \sin A \)

4 STATISTICS

Mean and standard deviation

Ungrouped data
\[
\text{Mean } (\bar{x}) = \frac{\sum x}{n}, \quad \text{SD} = \sqrt{\frac{\sum (x - \bar{x})^2}{n}} = \sqrt{\left\{ \frac{\sum x^2}{n} - (\bar{x})^2 \right\}}
\]

Grouped data
\[
\text{Mean } (\bar{x}) = \frac{\sum fx}{\sum f}, \quad \text{SD} = \sqrt{\frac{\sum f(x - \bar{x})^2}{\sum f}} = \sqrt{\left\{ \frac{\sum fx^2}{\sum f} - (\bar{x})^2 \right\}}
\]
1 Solve the simultaneous equations
\[ 2a + b - c = 5, \]
\[ a + 4b + 2c = 16, \]
\[ 15a + 6b - 2c = 12. \]  

2 (a) The surface area of a rectangular block of ice, \( A \text{cm}^2 \), is given by \( A = 6x^2 + \frac{216}{x} \). Find an expression for \( \frac{dA}{dx} \).

(b) Given that the block of ice in (a) is melting in such a way that \( A \) is decreasing at a constant rate of \( 0.14 \) cm\(^2\)/s, calculate the rate of decrease of \( x \) at the instant when \( x = 4 \).

3 (a) Find the range of values of \( x \) for which \( 2x^2 - x - 1 > 2x + 1 \).

(b) Express \( 2x^2 - 5x + 16 \) in the form \( a(x - b)^2 + c \), where \( a, b \) and \( c \) are constants. Hence find the minimum value of \( 2x^2 - 5x + 16 \).

4 Solve the equations

(a) \( 5^{x-1} = 13 \).

(b) \( \log(2x) - \log(x - 3) = 1 \).

5 The expression \( f(x) = cx^3 + 8x^2 + dx + 6 \) is exactly divisible by \( x^2 - 2x - 3 \).

(a) Find the value of \( c \) and \( d \).

(b) Find the remaining factor of the expression.

(c) If \( f(x) \) is not divisible by \( x + 2 \), find the remainder.

6 (a) In how many ways can 7 red marbles and 3 green marbles be put in a straight line, if

(i) there are no restrictions,

(ii) green marbles should not be next to each other?

(b) A group of 6 pupils is to be chosen from 10 boys and 8 girls. Find the number of ways of choosing at least 4 boys.
7 Find all the angles between $0^\circ$ and $360^\circ$ which satisfy the equation:

(a) $\tan \theta = -1.333$, 
(b) $\cos 3y = -0.5$, 
(c) $3\cos^2 z + 4\sin z = 4$. 

8 A pupil was given a 60 cm piece of wire to design a rectangular box for storing pens. The designer came up with the following diagram measuring 4x cm by x cm by y cm.

![Diagram of a rectangular box]

Given that the whole piece of wire was used,

(a) express the length of y in terms of x. 
(b) find an expression for the volume enclosed by the framework in terms of x. 
(c) calculate the value of x which makes the volume stationary. 
(d) show that this volume is maximum and find this volume.

9 (a) The third term of a geometric progression of positive terms is 18 and the fifth term is 162.

(i) Find the common ratio and the first term. 
(ii) Write an expression for the $n^{th}$ term of the progression. 
(iii) Find the sum of the first eight terms of the progression.

(b) Find the geometric mean of $\frac{1}{64}$ and $\frac{1}{16}$.

(c) Find the number of terms of the arithmetic progression 15, 19, 23,... that must be taken for the sum to be equal to 444.

10 The table below shows the frequency with which words of various lengths appear in the first paragraph of a particular book.

<table>
<thead>
<tr>
<th>No. of letters</th>
<th>1 - 3</th>
<th>4 - 6</th>
<th>7 - 9</th>
<th>10 - 12</th>
<th>13 - 15</th>
</tr>
</thead>
<tbody>
<tr>
<td>No. of words</td>
<td>35</td>
<td>33</td>
<td>34</td>
<td>13</td>
<td>5</td>
</tr>
</tbody>
</table>

Find the estimate of

(a) the mean, 
(b) the variance, 
(c) the standard deviation.
11  A particle starts from rest at a point O and moves in a straight line with velocity \( v \) m/s given by \( v = 6t - 3t^2 \), where \( t \) is the time in seconds after the start.

Find
(a)  the acceleration of the particle when it is next at instantaneous rest, \([3]\]
(b)  the maximum velocity, \([3]\]
(c)  the distance travelled to that position when the particle was next at instantaneous rest. \([4]\]

12  Answer only one of the following alternatives:

EITHER

The curve \( y = e^{-2x} - 2 \) meets the x-axis at A and the y-axis at B.

(a)  Find the coordinates of A and B. \([3]\]
(b)  Sketch the graph of \( y = e^{-2x} - 2 \) for the domain \(-1 \leq x \leq 1\). \([3]\]
(c)  Find the equation of the straight line which must be drawn on the graph of \( y = e^{-2x} - 2 \) to obtain a solution of the equation \( x = \ln\left(\frac{1}{\sqrt{2x+3}}\right)\). \([3]\]
(d)  State the range of \( f: x \rightarrow e^{-2x} - 2 \). \([1]\]

OR

(a)  Grace would like to give a sum of money to a charity each year for 10 years. She decides to give $120 in the first year, and to increase her contribution by $10 each year.

(i)  How much does she give in the last year? \([3]\]
(ii)  How much does the charity receive from her altogether? \([3]\]

(b)  A geometric series has first term 27 and common ratio \( \frac{4}{3} \). Find the least number of terms the series can have if its sum exceeds 729. \([4]\]